Int. J. Multiphase Flow Vol. 10, No. 2, pp. 149–157, 1984
Printed in Great Britain.

0301-9322/84 $3.00 + .00
Pergamon/Elsevier

TWO-DIMENSIONAL APPROACH TO THE MOTION OF A RED BLOOD CELL IN A PLANE COUETTE FLOW OF PLASMA

MASAKO SUGIHARA and HIDEYUKI NIIMI
National Cardiovascular Center Research Institute, Suita, Osaka, 565 Japan

(*Received* 8 *June* 1983; *in revised form* 1 *September* 1983)

Abstract—In relation to microrheology of blood, a theoretical approach to the motion of a red blood cell in a plane Couette flow between two parallel plates is made with emphasis on effects of wall. The red blood cell is assumed to be an elliptic cylindrical particle with a thin, inextensible membrane moving like a tank-tread along its perimeter and to contain a Newtonian fluid inside. Fluid motions are analysed numerically both inside and outside the particle on the basis of the Stokes equations, using the finite element method.

A quasi-static equilibrium condition leads to the solution for the motion of the particle. It is shown that two types of motion exist (a stationary orientation motion and a flipping motion), depending on the viscosity ratio of inner to outer fluid, the axis ratio of the elliptic cylinder and the ratio of particle size to channel width. The results are applied to capillary blood flow.

1. INTRODUCTION

Blood is regarded as a suspension of red blood cells (RBCs) in plasma. The rheological properties of blood depend crucially on the mechanical properties of the RBCs. Thus, it is important to study the motion of a single RBC in shear flow of plasma for a good understanding of the bulk rheology of blood.

RBC consists of a thin membrane which encloses a Newtonian solution of hemoglobin and it may be deformed easily at a constant surface area. Due to its deformability, the motion of RBC in flow shows individual features. It is reported experimentally that RBC may exhibit two types of motion in a simple shear flow. At a low shear rate, the RBC undergoes a flipping motion, while at a high shear rate, the RBC exhibits a steady orientation motion with its membrane rotating uniformly along its perimeter (tank-tread motion) (Schmid-Schonbein & Wells 1969; Goldsmith & Marlow 1972; Fischer *et al.* 1978; Schmid–Schonbein & Gaehtgens 1981). Quite recently, it has been demonstrated clearly in in-vitro experiments that the RBC membrane could be in tank-tread motion in blood flow through capillaries (Gaehtgens 1981).

It is not suitable to assume RBC as a rigid particle or a liquid drop for describing the motion of RBC in flow. In fact, a rigid particle exhibits only the unsteady flipping motion in shear flow (Jeffery 1922; Happel & Brenner 1965). A liquid drop shows a non-uniform rotation of its surface, while the RBC membrane moves uniformly due to its inextensibility.

The transition between the two types of RBC motion in a simple shear flow has been studied theoretically by several investigators. Kholeif & Weymann (1974) adopted a two-dimensional model for RBC, and Keller & Skalak (1982) adopted a three-dimensional model. In their theoretical approaches, the RBC is assumed to contain a Newtonian fluid inside and to be bounded by an inextensible thin membrane that allows to have a tank-treading motion. Their results provide theoretical evidences to the existence of both stationary orientation motion and unsteady flipping motion in a simple shear flow.

It is not doubtful that RBCs may interact with each other in blood flow. In addition, vessel wall may affect the motion of RBCs near it. Therefore, it is an important task to study how RBCs interact with each other or with a wall for blood flows in capillaries or near vessel wall (Sugihara & Niimi 1983). The motion of a single RBC in shear flow has not been clarified under the cell–cell interaction or the cell-wall interaction yet.

In this paper, we make a numerical approach to the motion of a RBC in a plane

Couette flow with emphasis on effects of wall. The RBC is modelled as a cylindrical particle with an elliptic cross-section which has a thin, inextensible flexible membrane and contains a Newtonian fluid inside; its shape is prescribed as an independent entity. The only allowed motion of the membrane is along the perimeter of the cell such that the shape of the cell is not changed. The particle is suspended in another Newtonian fluid and placed at the centerline between two parallel plates moving along themselves to opposite direction at a speed.

It is assumed that inertial effects are neglected so that the Stokes equations govern the fluid motions both inside and outside the particle. The motion of the particle is determined under quasi-static equilibrium conditions, using the finite element method in terms of the primitive variables, such as velocities and pressure (Olson & Tuann 1978).

2. FORMULATION

We consider the motion of a model RBC in a plane Couette flow between two parallel plates. The coordinates system (x, y) may be so chosen that the center of a model RBC is at the origin, and the parallel plates are $y = \pm d$ (figure 1). The velocity of the upper plate is taken to be U_w in the x-direction while that of the lower plate is $-U_w$.

The model RBC is assumed to be an elliptic cylindrical particle with its semi-axes a and b ($\eta = a/b \geq 1$); it contains an incompressible Newtonian fluid of viscosity μ^i inside and the membrane moving uniformly along its perimeter.

The suspending medium is an incompressible Newtonian fluid of viscosity μ^e. Since the fluid motion both inside and outside the particle is assumed to obey the Strokes equations, the following quasi-static equilibrium conditions are satisfied: (1) The torque acting on the particle vanishes. (2) The resultant tangential force acting on the membrane also vanishes. Note that the resultant drag and lift acting on the particle always vanish as the particle is placed at the centerline of the plates. Under the conditions (1) and (2), the motion of the particle is determined together with the stresses acting on both sides of the membrane.

In the numerical calculations, we use the finite element method in terms of velocities and pressure; the domain of interest is divided into a finite number of triangular elements. The shape of the particle is approximated by a polygon inscribed in its elliptic cross-section. The velocity components and the pressure within each element are made polynomial approximation in space variables. Then, simultaneous equations for velocity components and pressure at the nodes obtained on the basis of variational principles are solved to determine the flow fields.

External flow

The undisturbed flow is expressed as $\mathbf{U}_0 = (ky, 0)$ where $k = U_w/d$. The angle between the major axis of particle and the x-axis is denoted by θ; its positive value means a counterclockwise rotation.

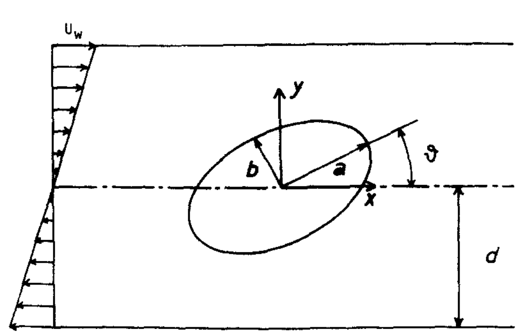

Figure 1. Flow configurations.

The velocity fields must satisfy the two-dimensional Strokes equations and the continuity equation:

$$\frac{\partial p}{\partial x} = \mu^e \nabla^2 u, \quad \frac{\partial p}{\partial y} = \mu^e \nabla^2 v, \qquad [2.1]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \text{ and } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \qquad [2.2]$$

where p is the pressure and $\mathbf{u} = (u, v)$ is the velocity vector.

Far from the particle, the velocity field is assumed to approach to a plane Couette flow $\mathbf{U}_0$. On the surface of the particle, non-slip condition is adopted. Then, the boundary conditions are

$$\mathbf{u} = (U_w, 0) \quad \text{at } y = d, \qquad [2.3]$$

$$\mathbf{u} = (-U_w, 0) \quad \text{at } y = -d, \qquad [2.4]$$

$$\mathbf{u} = \mathbf{t}U_m + \mathbf{e}\Omega \qquad [2.5]$$

on the surface of the particle, and

$$\mathbf{u} = \mathbf{U}_0 \qquad [2.6]$$

far from the particle,

where $\mathbf{e} = (-y, x)$, $\mathbf{t}$ is unit vector (counterclockwise) tangent to the membrane. The quantities U_m and Ω represent the tank-treading velocity of the membrane and the angular velocity of the particle, respectively; their values are to be determined later as part of the solution. In numerical calculation, we apply the condition [2.6] at $x = \pm l$, where l is taken sufficiently greater than d.

To solve this problem, we consider three cases: (i) the stationary rigid particle in plane Couette flow ($U_m = 0$, $\Omega = 0$), (ii) the stationary particle with its membrane tank-treading in an otherwise quiescent fluid ($U_w = 0$, $\Omega = 0$), and (iii) the rigid particle flipping with an angular velocity in an otherwise quiescent fluid ($U_w = 0$, $U_m = 0$). If we superimpose these cases, the solution for the problem can be obtained since the basic equations are linear. For each case, the stresses acting on the membrane are calculated to determine both the torque T^e and the tangential force $F_m^{\ e}$.

Internal flow

The flow fields are determined under the boundary condition:

$$\mathbf{u} = \mathbf{t}U_m \qquad [2.7]$$

on the surface of the particle.

The stresses acting on the membrane are calculated to obtain the torque T^i and the tangential force $F_m^{\ i}$.

The motion of the particle

It is possible to examine the motion of the particle from the previous results. We can determine U_m and Ω under the quasi-static equilibrium conditions, as functions of three parameters: the viscosity ratio $\nu(=\mu^i/\mu^e)$, the axis ratio $\eta(=a/b$; shape factor), and the ratio $\lambda(=\sqrt{ab}/d$; size factor).

3. RESULTS AND DISCUSSION

We have calculated the torque T^e and the tangential force $F_m^{\ e}$ acting on the membrane for the external flow in three cases (i), (ii) and (iii). The coefficients $C_T^{\ e}$, $C_F^{\ e}$ are defined by $-T^e/\mu^e U d$ and $-F_m^{\ e}/\mu^e U$, respectively, where U means U_w, U_m and $\Omega\sqrt{ab}$ for cases (i), (ii) and (iii), respectively. These coefficients are even and periodic functions of θ with a period π. In figure 2 are shown the coefficients $C_T{}^e$ and $C_F{}^e$ against θ for $\eta = 1.44$, $\lambda = 0.5$. In case (i), $C_T{}^e$ increases as θ increases from 0 to $\pi/2$, while $C_F{}^e$ becomes smaller as θ becomes larger. Clearly, $C_T{}^e$ in case (ii) and $C_F{}^e$ in case (iii) are almost constant, but increasing θ increases $C_F{}^e$ in case (ii) and decreases $C_T{}^e$ in case (iii).

For internal flow, T^i and $F_m{}^i$ are calculated for various values of η. In figure 3 are plotted the coefficient $C_T{}^i$ $(= -T^i/\mu^i U_m\sqrt{ab})$ and $C_F{}^i$ $(= -F_m{}^i/\mu^i U_m)$ against η. Note that both of the coefficients are very small for $\eta = 1$, corresponding to the particle of circular cross-section.

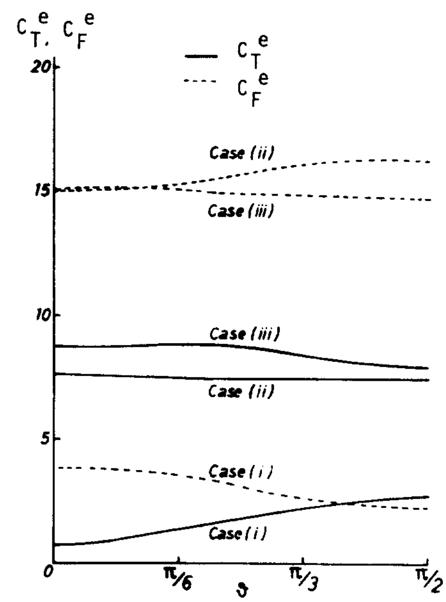

Figure. 2. Coefficients $C_T{}^e$ and $C_F{}^e$ vs angle of inclination θ for external flow ($\eta = 1.44$, $\lambda = 0.5$).

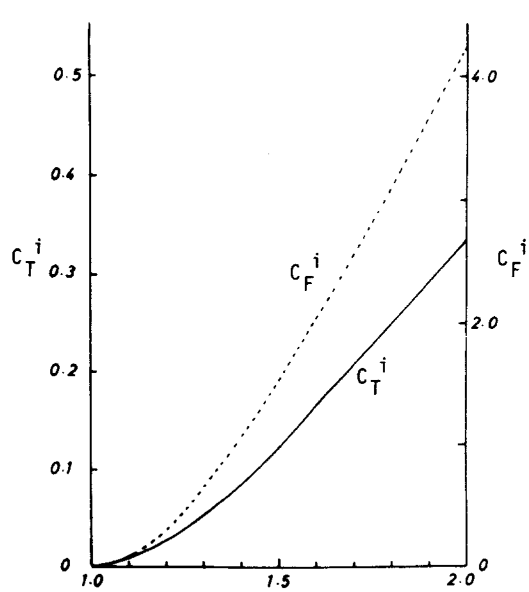

Figure 3. Coefficients $C_T{}^i$ and $C_F{}^i$ vs axis ratio η for internal flow.

We superimpose the solutions for external flow in three cases and for internal flow to apply the conditions:

$$\Sigma T^e + T^i = 0, \ \Sigma F_m^e + F_m^i = 0, \qquad [3.1]$$

Σ referring to the summation of three cases. Then, we can calculate the tank-treading velocity U_m and the angular velocity Ω. Figure 4 shows U_m/U_w and Ω/k vs θ for $\eta = 1.44$, $\lambda = 0.5$ and several values of v. It is interesting to note that Ω decreases monotonically with increasing θ. Especially, for v below v_c (approx. 21), there exists θ^* of θ where Ω becomes zero; θ^* falls between 0 and $\pi/4$. Let us call the angle θ^* as stationary orientation angle and denote the velocity U_m at θ^* by U_m^*. Since Ω corresponds to change of θ in time, θ increases in time within its range smaller than θ^*, where Ω is positive. On the contrary, θ decreases in time within its range larger than θ^*. Consequently, the angle of inclination θ approaches in time asymptotically to the stationary angle θ^*. Note that the tank-treading velocity U_m^* is negative. These results indicate that for v below v_c the particle undergoes the stationary orientation motion at the angle of inclination θ^* with membrane tank-tending in clockwise sense. In the range v below v_c, increasing v decreases θ^* and U_m^*. Especially, θ^* becomes zero at $v = v_c$. It is interesting to note that an increase in viscosity ratio decreases the tank-treading velocity in the stationary orientation motion. Figure 5 shows the velocity vectors of medium near the stationary orientation motion of the particle for $\eta = 1.44$, $\lambda = 0.5$ and $v = 3.6$ ($\theta^* = \pi/6$, $U_m^*/U_w = -0.18$).

The angular velocity Ω is always negative when v is larger than v_c (figure 4); this indicates that the particle is periodically flipping in clockwise sense. Furthermore, Ω is small near $\theta = 0$ but large near $\theta = \pi/2$, suggesting that the particle spends a greater proportion of the period of motion with its major axis aligned with the flow direction.

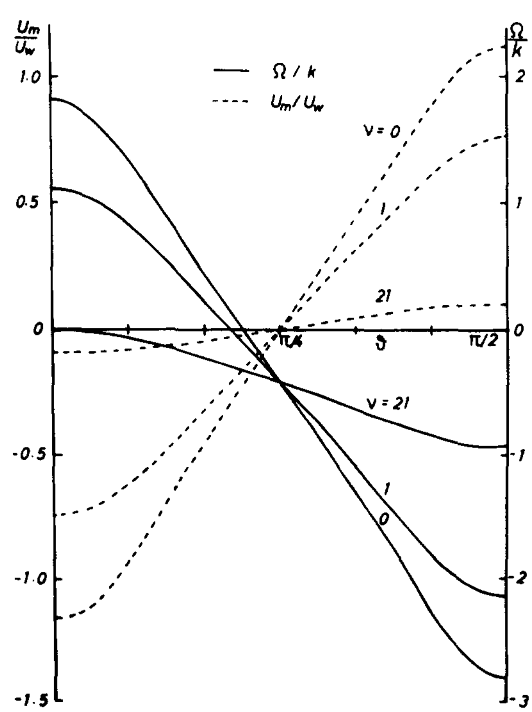

Figure 4. Tank-treading velocity U_m/U_w and angular velocity Ω/k vs angle of inclination θ for $\eta = 1.44$ and $\lambda = 0.5$.

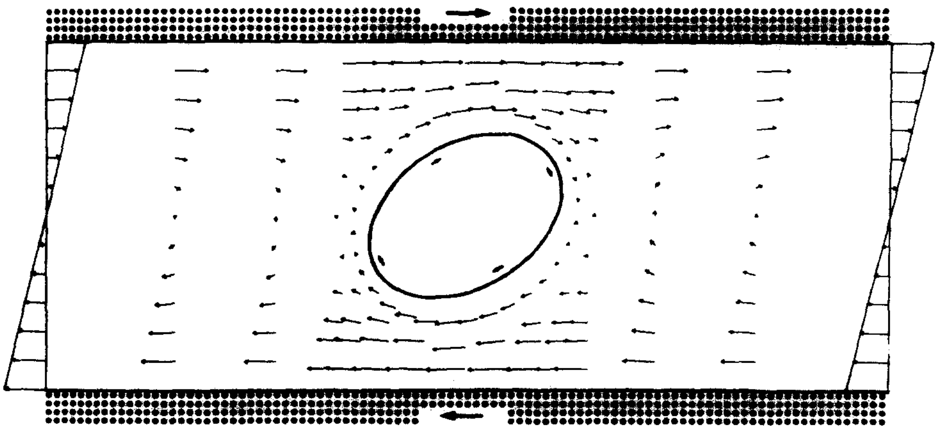

Figure 5. Velocity vectors of medium around the particle in stationary orientation motion for $\nu = 3.6$, $\eta = 1.44$ and $\lambda = 0.5$ where $\theta^* = \pi/6$ and $U_m^*/U_w = -0.18$.

The stationary angle θ^* depends on the viscosity ratio ν, the shape factor η and the size factor λ. In figure 6 is plotted θ^* as a function of η for $\lambda = 0.5$ and several values of ν. It is seen that for a constant η, θ^* decreases as ν increases from 0 to ν_c until θ^* vanishes at $\nu = \nu_c$. In addition, θ^* decreases monotonically with an increase in η for a small value of ν, while θ^* comes to minimum at a value of η for a large value of ν. Figure 7 is a plot of θ^* vs λ for $\eta = 1.44$. The angle θ^* decreases with increasing λ for a value of ν near or smaller than 1. For ν larger than its value, the angle θ^* increases with increase in λ.

It is obtained that two types of motion of the particle occur depending on the values of ν, η and λ (the stationary orientation motion and the flipping motion). In figure 8 is plotted the transition value ν_c of ν vs η for three values of λ. The $\eta - \nu_c$ curves distinguish the two types of motion; the upper region corresponds to the flipping motion, while the lower to the stationary orientation motion. Since the $\eta - \nu_c$ curve has a minimum point

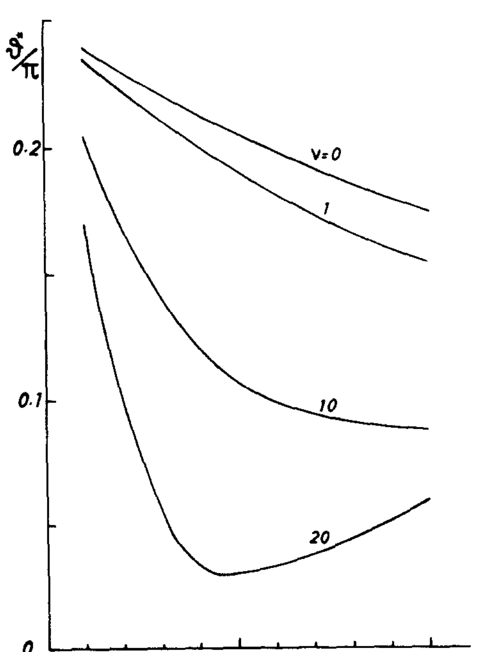

Figure 6. Stationary angle of inclination θ^* vs axis ratio η for $\lambda = 0.5$.

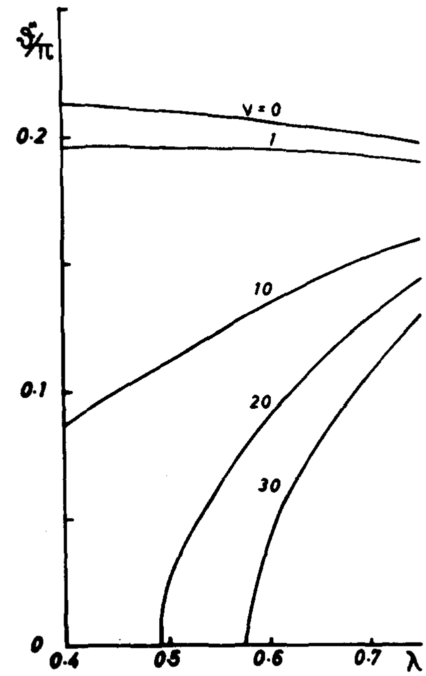

Figure 7. Stationary angle of inclination θ^* vs size factor λ for $\eta = 1.44$.

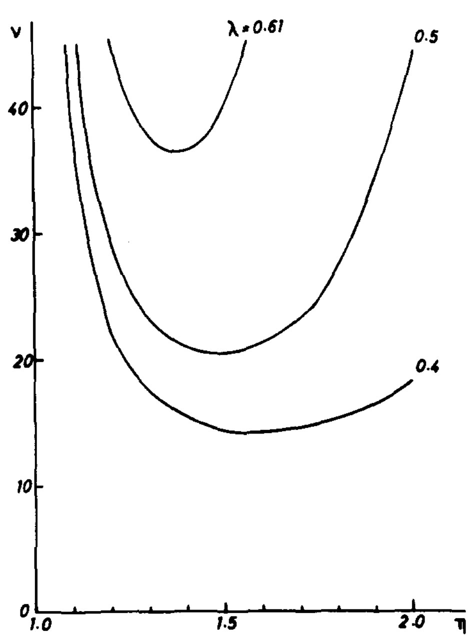

Figure 8. Transition value of viscosity ratio ν vs axis ratio η for $\lambda = 0.4$, 0.5 and 0.61.

at a value of η (say η_c) between 1 and 2, the particle transits from the flipping motion to the stationary orientation motion as η becomes away from η_c with a constant value of ν. If we consider the relationship between ν_c and λ through the intersections of η = constant and a family of $\eta - \nu_c$ curves, we see that the transition value ν_c increases with increasing λ. This indicates that the larger particle has a tendency to exhibit the stationary orientation motion.

4. CONCLUDING REMARKS

The motion of the RBC model in a Couette flow between two parallel plates has been analysed numerically using the finite element method. This method is very useful for treating complicated body-shapes. In our formulation, the primitive variables have been used, because they are more physical and have lower order than other variables such as stream functions.

The present results indicate that the motion of the particle may exhibit the transition between a stationary orientation motion and an unsteady flipping motion, depending on the three parameters: viscosity ratio ν, shape factor η and size factor λ. Decreasing viscosity ratio or increasing size factor promotes the stationary orientation motion, with the opposite variations inducing the flipping motion. For constant ν and λ, the particle with shape factor η_c has a tendency to exhibit the flipping motion. For η beyond η_c, increasing η favours the stationary orientation motion.

Our results have been obtained in the two-dimensional model, but they give some insight into blood flow. The numerical results have demonstrated how the wall may promote the particle in a stationary orientation motion. This provides a theoretical evidence to the stationary orientation motion of RBCs with the membrane tank-tread observed in an extremely narrow cone-plate "rheoscope" (5 μm and 15 μm width) (Fischer *et al.* 1978).

In the present analyses, the shape of the particle is assumed to be prescribed without any change in motion. If the particle is deformable, it must be determined under the influence of stresses acting on its membrane. In addition, the particle experiences neither drag nor lift since it is placed at the centerline between two parallel plates. If the particle is placed off-center, it will experience some drag or lift to translate with tank-treading and/or flipping. Thus, the motion of the deformable particle in flow through a narrow channel is left for further study.

Acknowledgements—The authors would like to express their hearty thanks to Professors T. Tatsumi, K. Gotoh and Y. Sone for their valuable discussions. This paper was supported by a Grant-in-Aid for Fundamental Scientific Research from the Ministry of Education, Science and Culture, and a Research Grant for Cardiovascular Diseases from the Ministry of Health and Welfare.

A part of this work was presented at the 1st China–Japan–U.S.A. Conference on Biomechanics, on 9–12 May at Wuhan in China.

REFERENCES

Fischer, T. M., Stohr-Liesen, M. & Schmid-Schonbein, H. 1978 The red cells as a fluid droplet: Tank tread-like motion of the human erythrocyte membrane in shear flow. *Science* **202**, 894–896.

Gaehtgens, P. 1981 *In vitro* studies of blood rheology in microscopic tubes. In: *The Rheology of Blood, Blood Vessels and Associated Tissues* (Edited by D. R. Gross & N. H. C. Hwang), pp. 257–275. Sijthoff & Noordhoff, Amsterdam.

Goldsmith, H. L. & Marlow, J. 1972 Flow behavior of erythrocytes—I. Rotation and deformation in dilute suspensions. *Proc. Roy. Soc. Lond.* **B182**, 351–384.

Happel, J. & Brenner, H. 1965 *Low Reynolds Number Hydrodynamics*. Prentice-Hall, Englewood Cliffs, New Jersey.

Jeffery, G. B. 1922 The motion of ellipsoidal particle immersed in a viscous fluid. *Proc. Roy. Soc. Lond.* **A102**, 161–179.

Keller, S. R. & Skalak, R. 1982 Motion of a tank-treading ellipsoidal particle in a shear flow. *J. Fluid Mech.* **120**, 27–47.

KHOLEIF, I. A. & WEYMANN, H. D. 1974 Motion of a single red blood cell in plane shear flow. *Biorheology* **11**, 337–348.

OLSON, M. D. & TUANN, S. Y. 1978 Primitive variables versus stream function finite element solution of the Navier Stokes equation. In: *Finite Element in Fluids* (Edited by R. H. GALLAGHER, O. C. ZIENKIEWICZ, J. T. ODEN, M. MORANDI CECCHI & C. TAYLOR). Vol. 3, pp. 73–87. Wiley, New York.

SCHMID–SCHONBEIN, H. & WELLS, R. 1969 Fluid drop-like transition of erythrocytes under shear. *Science* **165**, 288–291.

SCHMID–SCHONBEIN, H. & GAEHTGENS, P. 1981 What is red cell deformability? *Scand. J. clin. Lab. Invest.* **41** (Suppl. **156**), 13–26.

SUGIHARA, M. & NIIMI, H. J. 1983 Asymmetric flow of a cylindrical particle through a narrow channel. *Trans ASME, Ser. E: J. Appl. Mech.* (in press).